Sediment entrainment by shallow flow

※Entrainment
地表面から土壌粒子が剥離されて
流水に取り込まれること

A few words on shear stress (τ)

（せん断応力）

τ = m.g.sinα = volume * density * downslope component of gravity

In 2 dimensions we can consider it as:

= area * density*downslope component of gravity

At a point:

$$\tau = \rho g h \sin\alpha$$

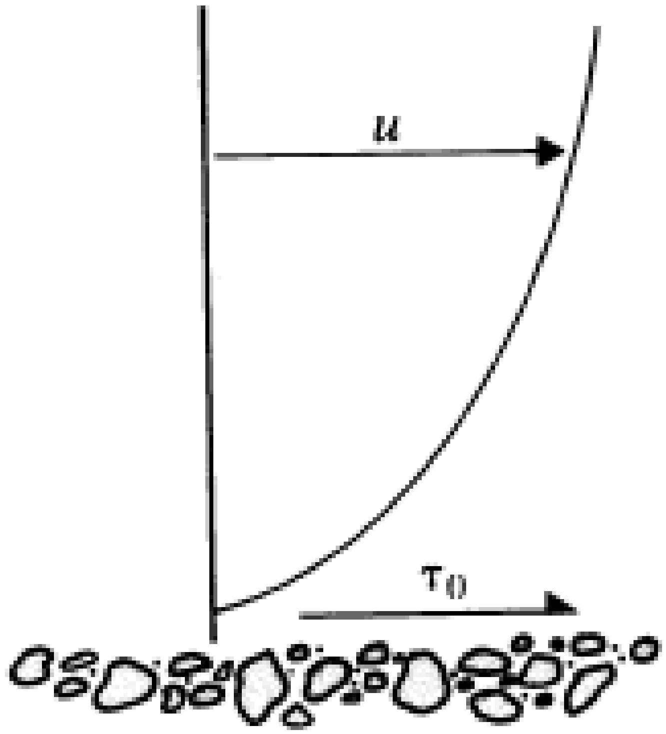
Where ρ (kg m^{-3}) is density,

g is gravity (m s^{-2}),

h is depth (m), and

α (m m^{-1}) is gradient

[Note: $u_* = \sqrt{(ghsin\alpha)}$] known as shear velocity （摩擦速度）
and has units m s^{-1}



(a) Shear force on granular bed

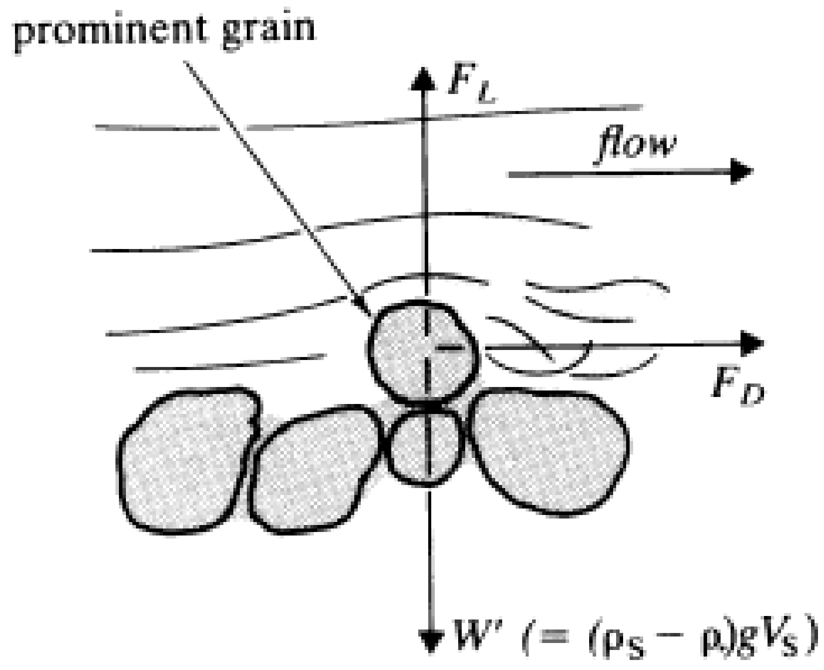
（粒状底面に作用するせん断力）

At a critical value of τ_o

（限界せん断応力）

widespread sediment

movement will begin



(b) Force on 'prominent' grain

（突出した粒子に作用する力）

The force F_D acting on a prominent particle is due to its separation of the flow pattern.

The number of prominent particles is related to the areal grain packing (A_p)

As the area of an individual particle is proportional to D^2, (where D is particle diameter) the number of prominent particles is a function of A_p/D^2

Since the total shear stress τ_0 is the sum of the shear stresses on all the particles, and most of the shear stress will be exerted on the prominent particles, the total force on each emergent particle can be expressed as

$$F_d \propto \tau_0 D^2 / A_p$$

（水中の）

The shear stress is resisted by the submerged weight of the particle($= (\rho_s - \rho_w) g \pi D^3 / 6$) for a spherical particle, and its angle of repose (φ) (angle of internal friction)

（球形粒子）

（安息角）

（内部摩擦角）

Hence, at the threshold of movement

$$\tau_{cr} \frac{D^2}{A_p} \propto (\rho_s - \rho_w) g \frac{\pi D^3}{6} \tan\varphi$$

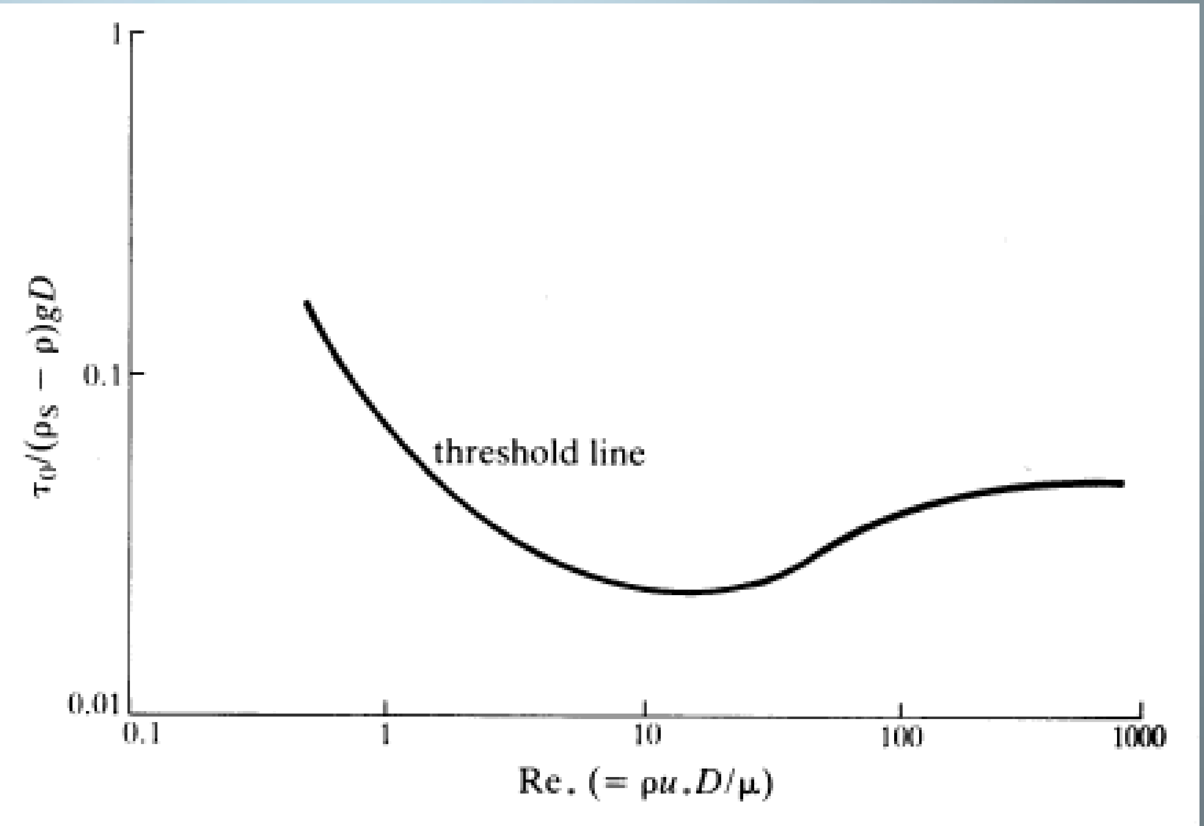
This can be re-arranged in a dimensionless form as

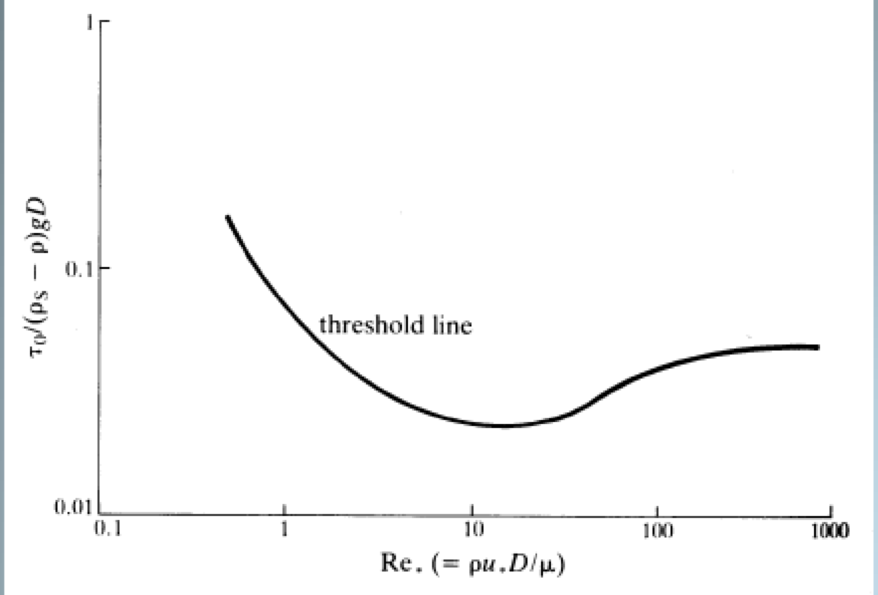
$$\frac{\tau_{cr}}{(\rho_s - \rho_w)gD} \propto \frac{\pi A_p}{6} \tan\varphi$$

in which the lefthand side is the entrainment function.

Shields reasoned that particle entrainment should be a function of Reynolds number
（レイノルズ数：流体の性質を表す）
and plotted the entrainment function against the Reynolds number*:

*at the grain





Beyond Re ~ 400 the value becomes a constant (=0.056), known as the Shields' number.

（シールズ数：底質の安定性を表す指標）

Hence, the Shields' equation

$$\frac{\tau_{cr}}{(\rho_s - \rho_w)gD} = 0.056$$

Sediment entrainment by water on hillslopes

Abrahams *et al*. (1988) proposed for shallow flow on hillslopes that:

$$\tau_c = 1227 D^{0.555} S^{1.094}$$

Cf Shields:

$$\frac{\tau_{cr}}{(\rho_s - \rho_w) g D} = 0.056$$

Nb: both are empirical – Shields ignores slope (he worked on low-angle slopes), and Abrahams ignores gravity and particle density (data come from a single site) and the equation doesn’t balance in terms of units

Diversion into the nature of overland flow on hillslopes

（浸透しきれない水の流出）

Horton (1933) – proposed the notion of infiltration-excess runoff. Subsequently termed Hortonian overland flow.

Basic idea:

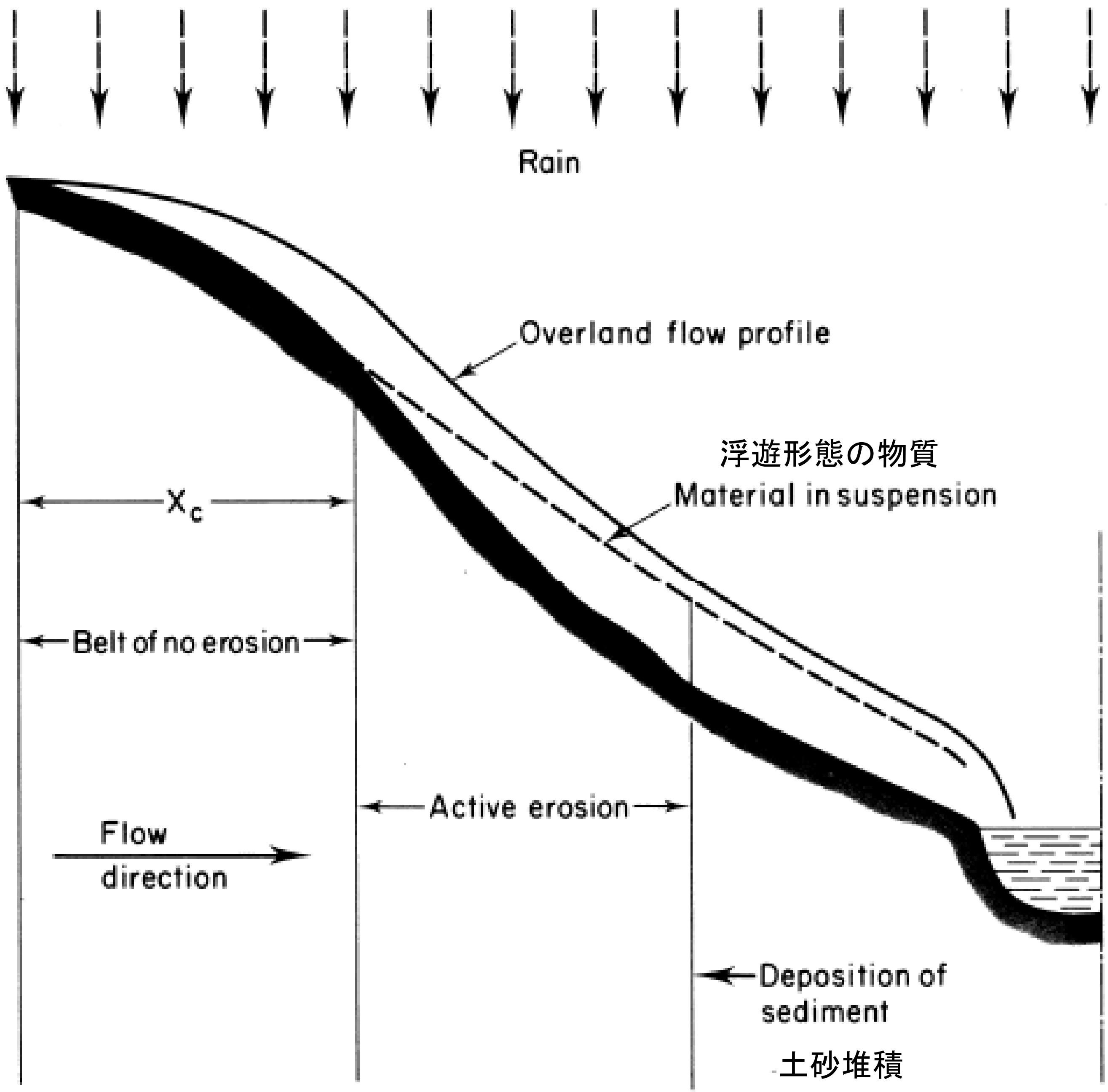
$$f = f_c + (f_0 - f_c)e^{-kt}$$

（浸透能）

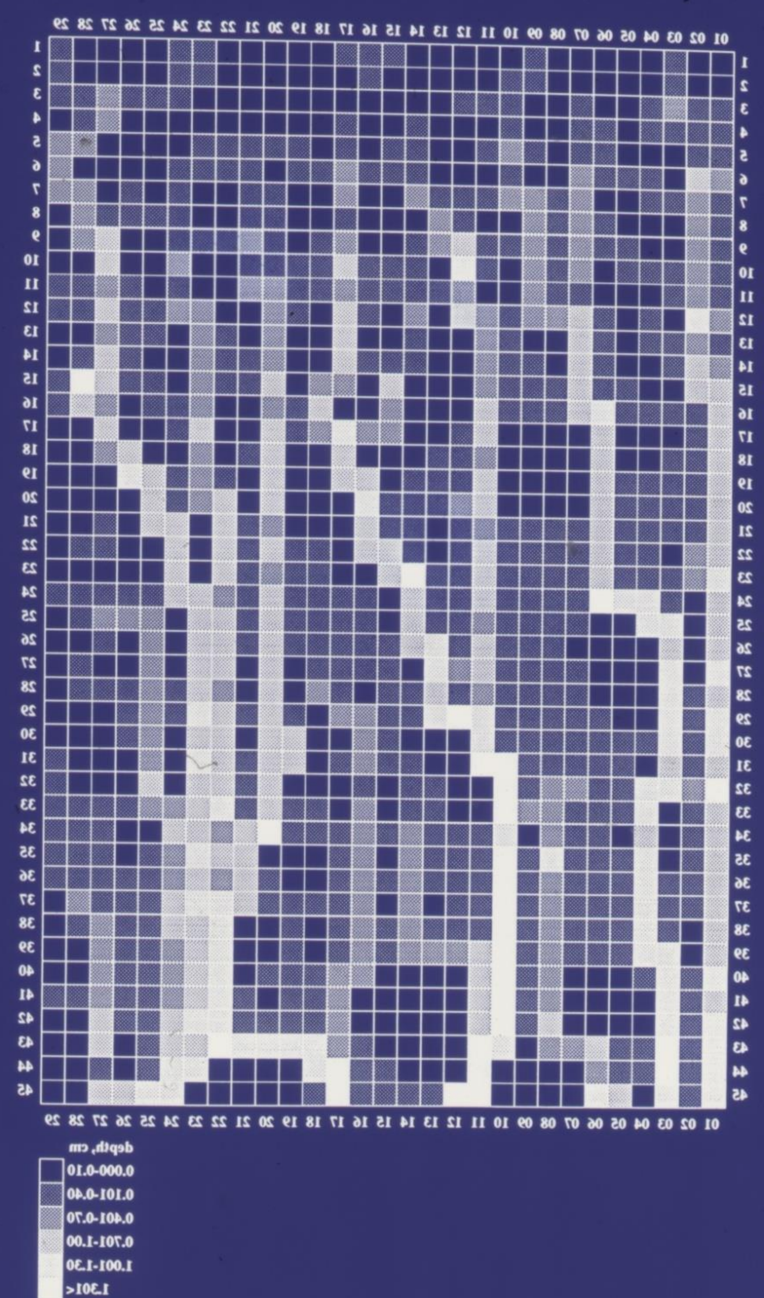
Where *f* is maximum instantaneous infiltration rate; f_c is the limiting steady infiltration rate, assumed to be a constant for a given soil type; f_0 is the initial infiltration rate at the start of the storm (t=0); *k* is a positive constant of permeability for a given soil; *t* is time

Model for Hortonian overland flow, therefore, is that:
（ホートン地表流）

- a) initially all rainfall infiltrates
- b) as rain continues infiltration rate declines
- c) once infiltration rate < rainfall rate water starts to pond at the surface and fill depressions (depression storage)
（地表面の窪地）
- d) depressions overtop and flow begins.

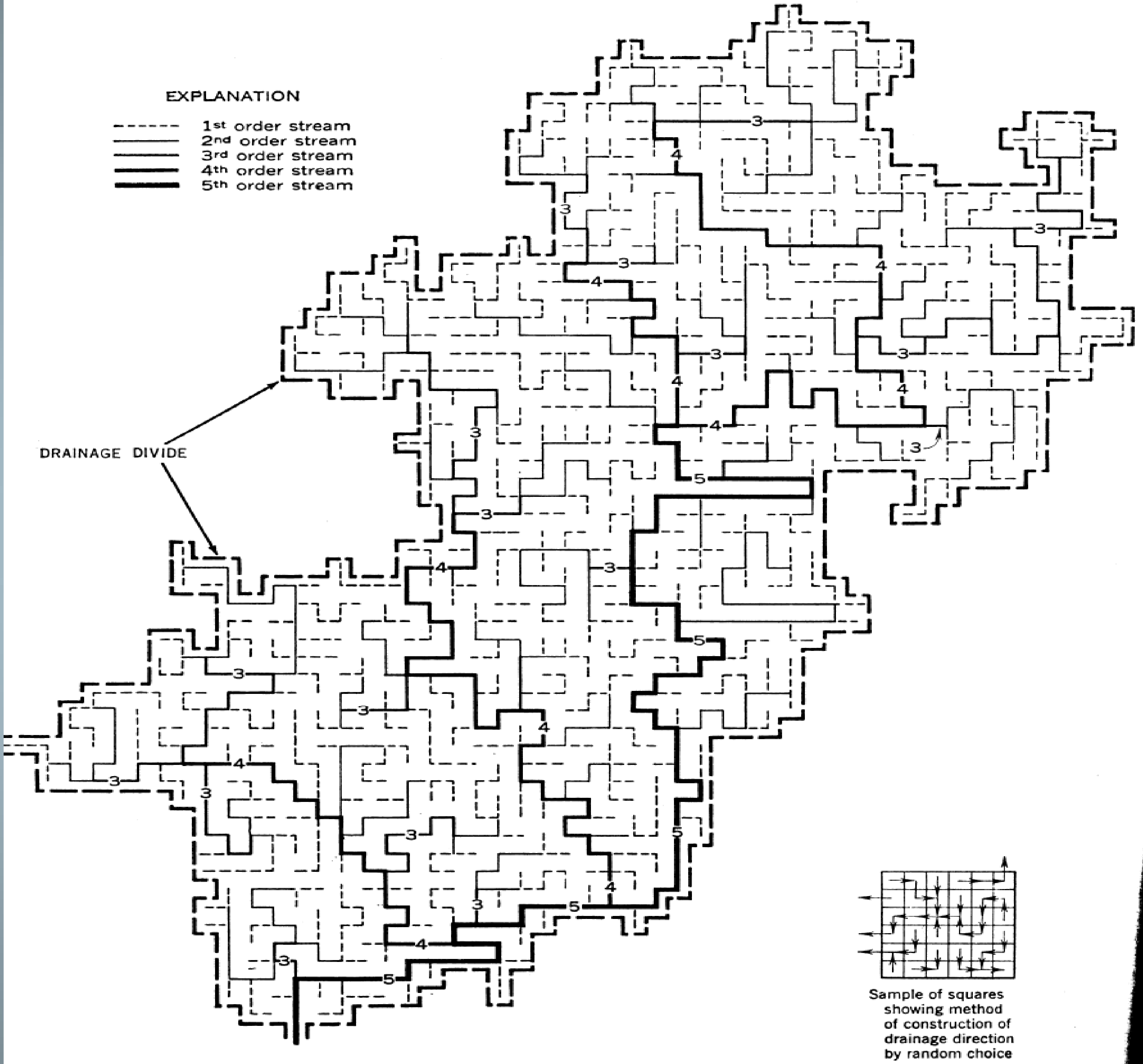

Rain
Overland flow profile
浮遊形態の物質
Material in suspension
X_c
Belt of no erosion
Active erosion
Flow
direction
Deposition of
sediment
土砂堆積

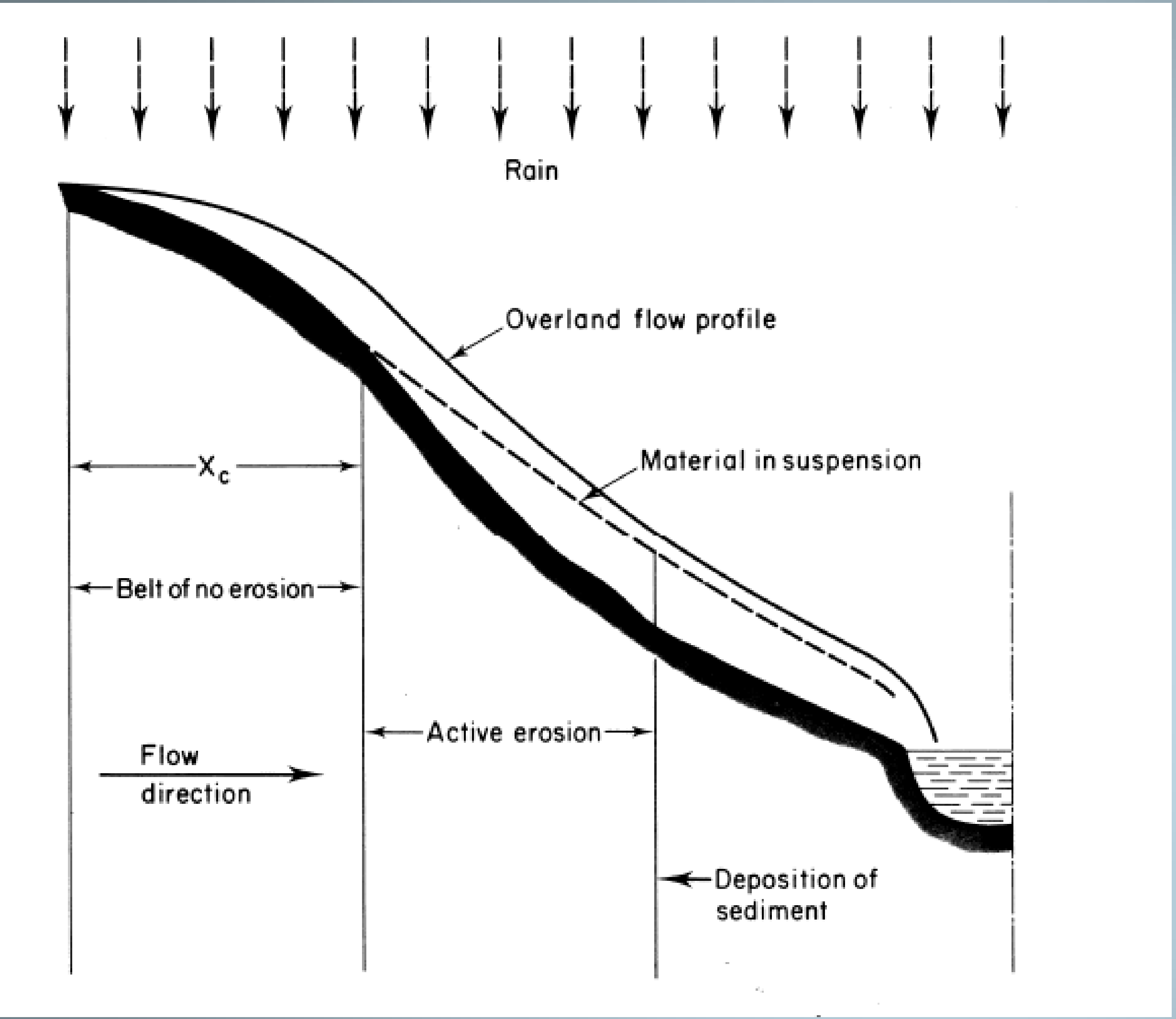
But overland flow isn't a sheet of water of uniform thickness



01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45
depth, cm
0.000-0.10
0.101-0.40
0.401-0.70
0.701-1.00
1.001-1.30
1.301<

Random walk model



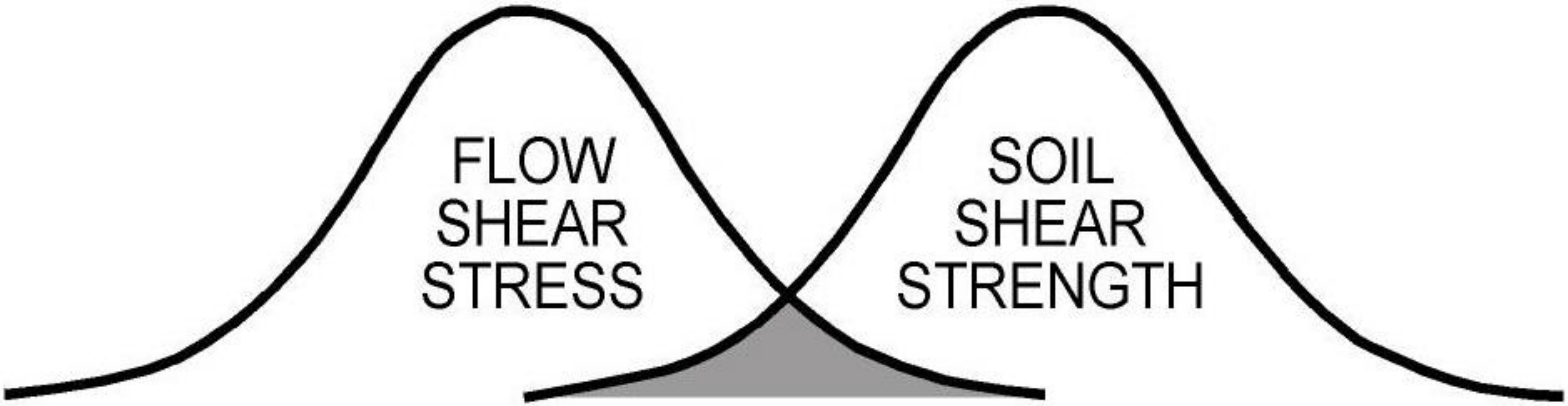


Horton's 'belt of no erosion' is incorrect, but studies have shown that detachment by flow does not occur in this zone.

Various authors (e.g. Govers, 1985; Bryan & Poesen 1989) have come to the conclusion that rills begin to form when flow shear velocity (= $u_* = \sqrt{ghs}$) is c. 3-3.5 cm s^{-1}

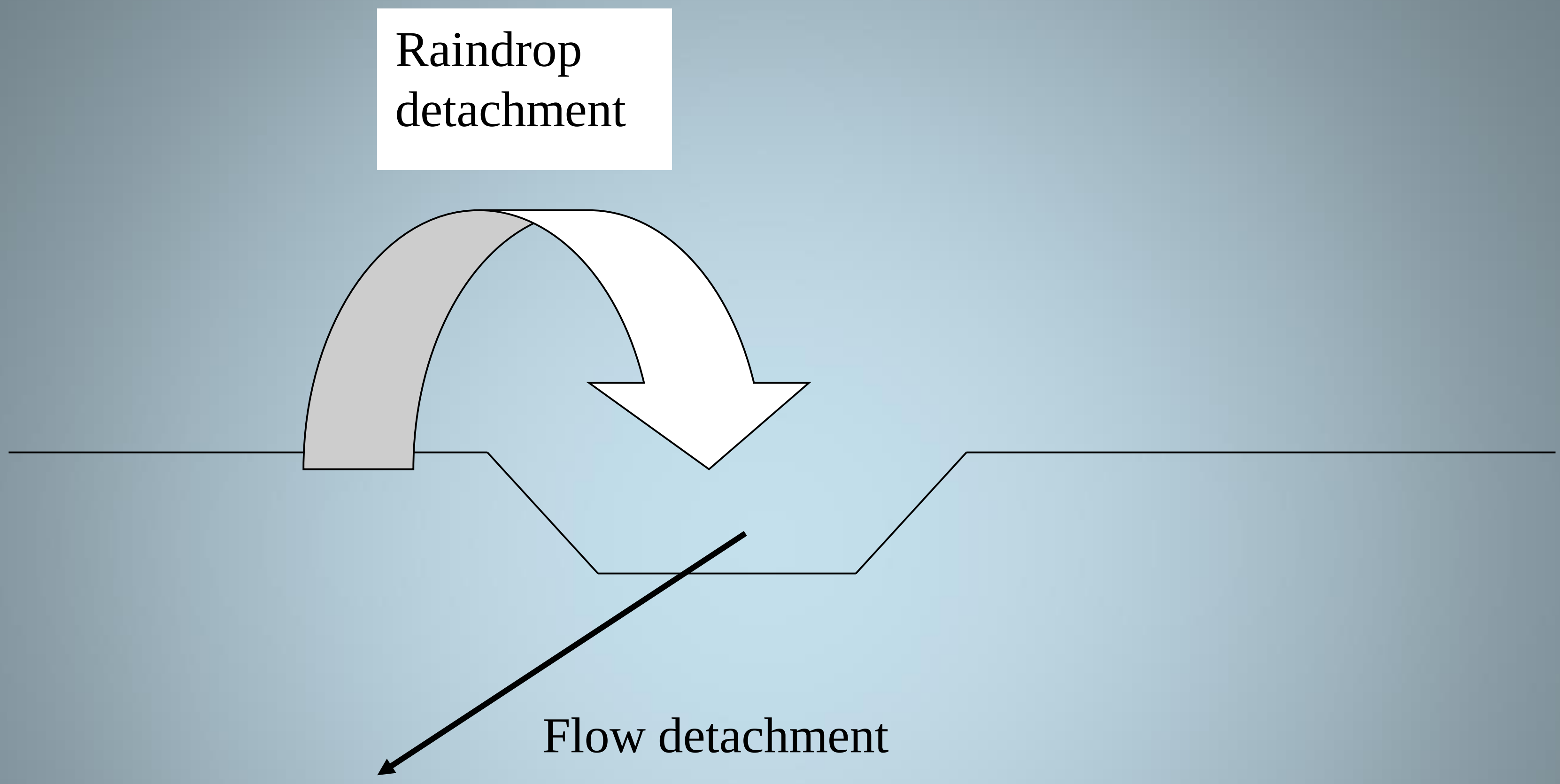
（せん断強度）

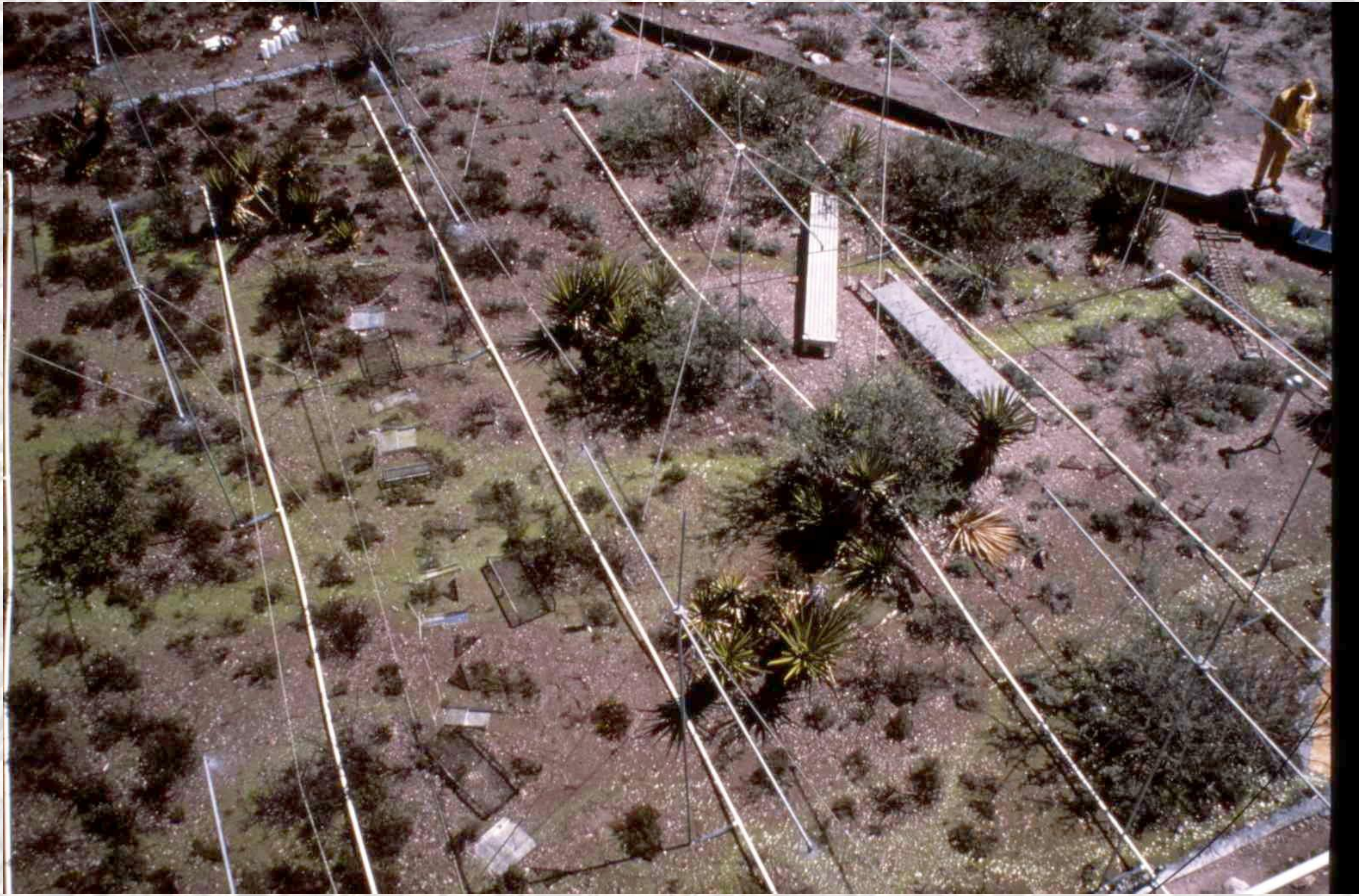
Problem is – soil shear strength (as conventionally measured) >> this (equivalent) shear stress

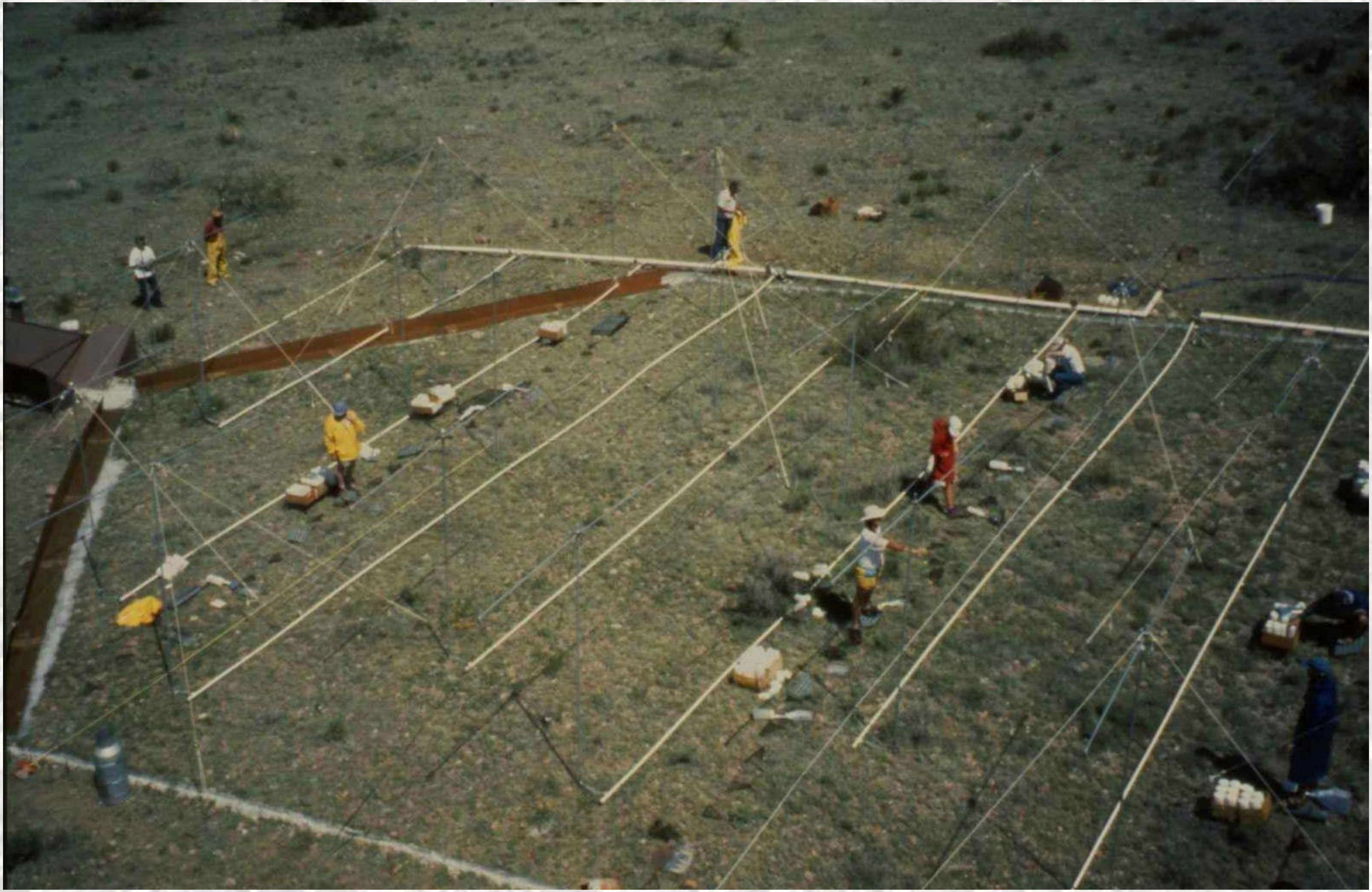

FLOW
SHEAR
STRESS
SOIL
SHEAR
STRENGTH

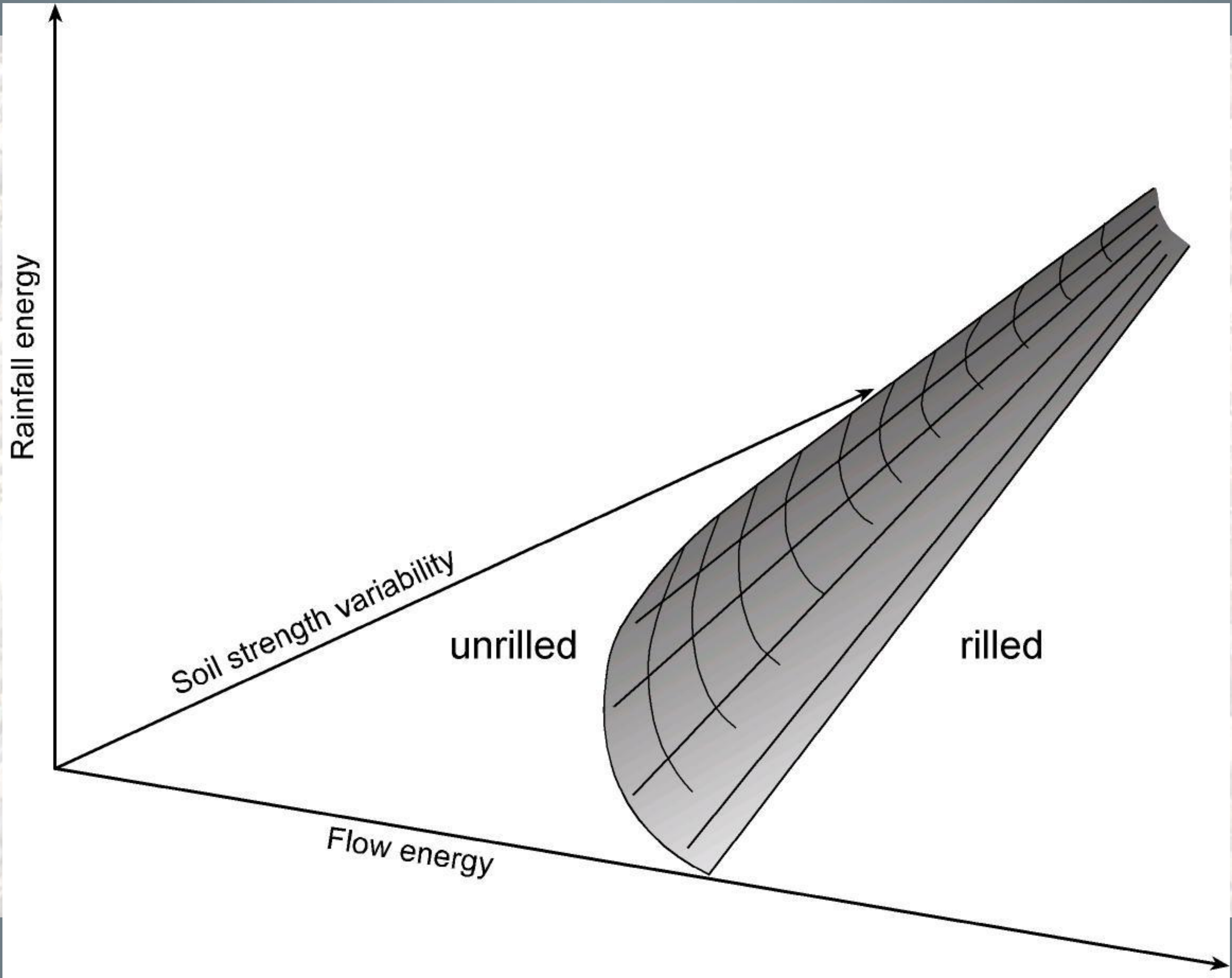
- **Conditions required for the formation of rills/sediment detachment by shallow flow:**

- 1. That there is a sufficiently high probability of obtaining turbulence in the distribution of flow depths
- 2. That there is a sufficiently high probability of local shear stress exceeding local shear strength
- 3. A suitable balance between raindrop detachment and flow detachment

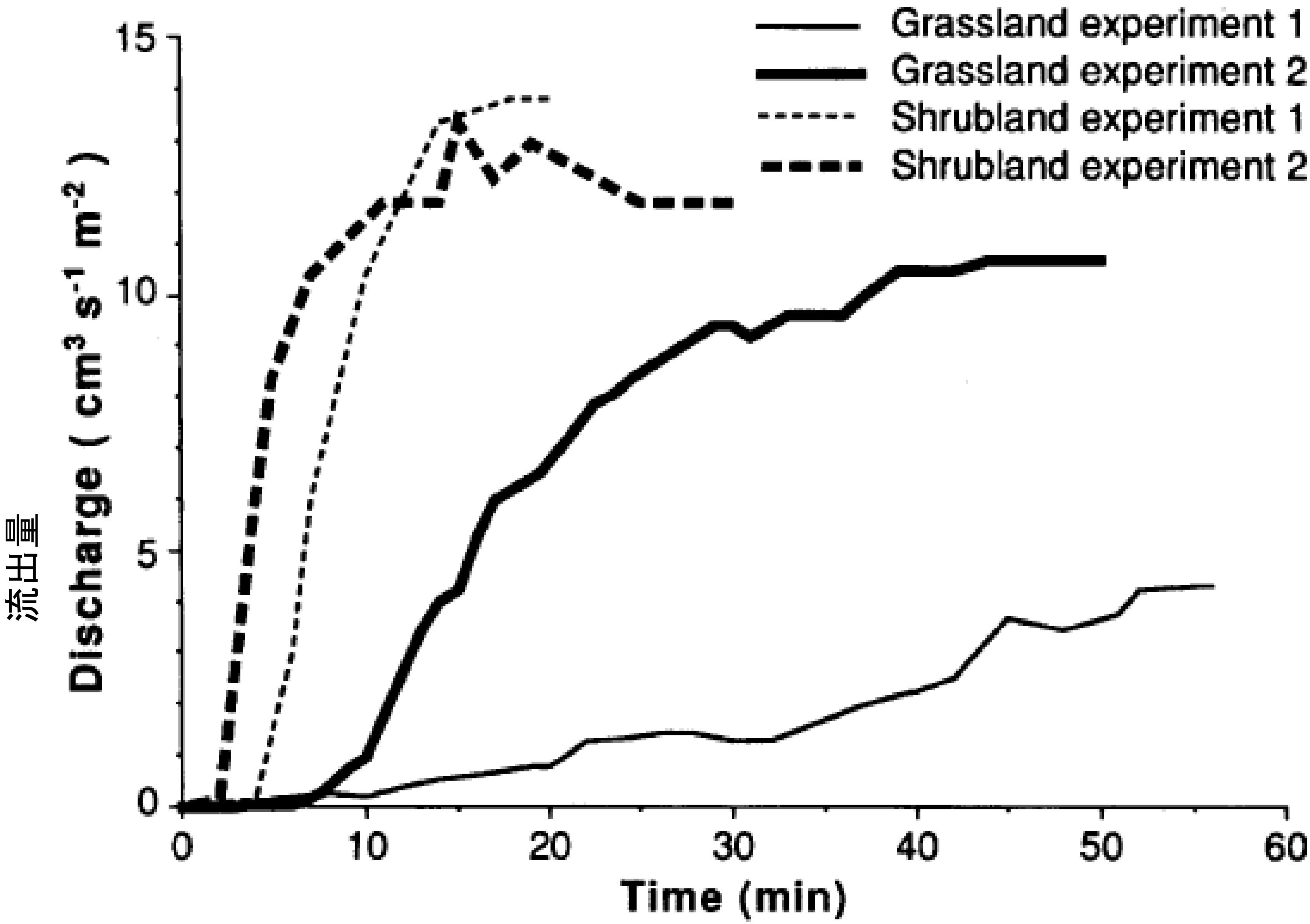

Raindrop
detachment
Flow detachment



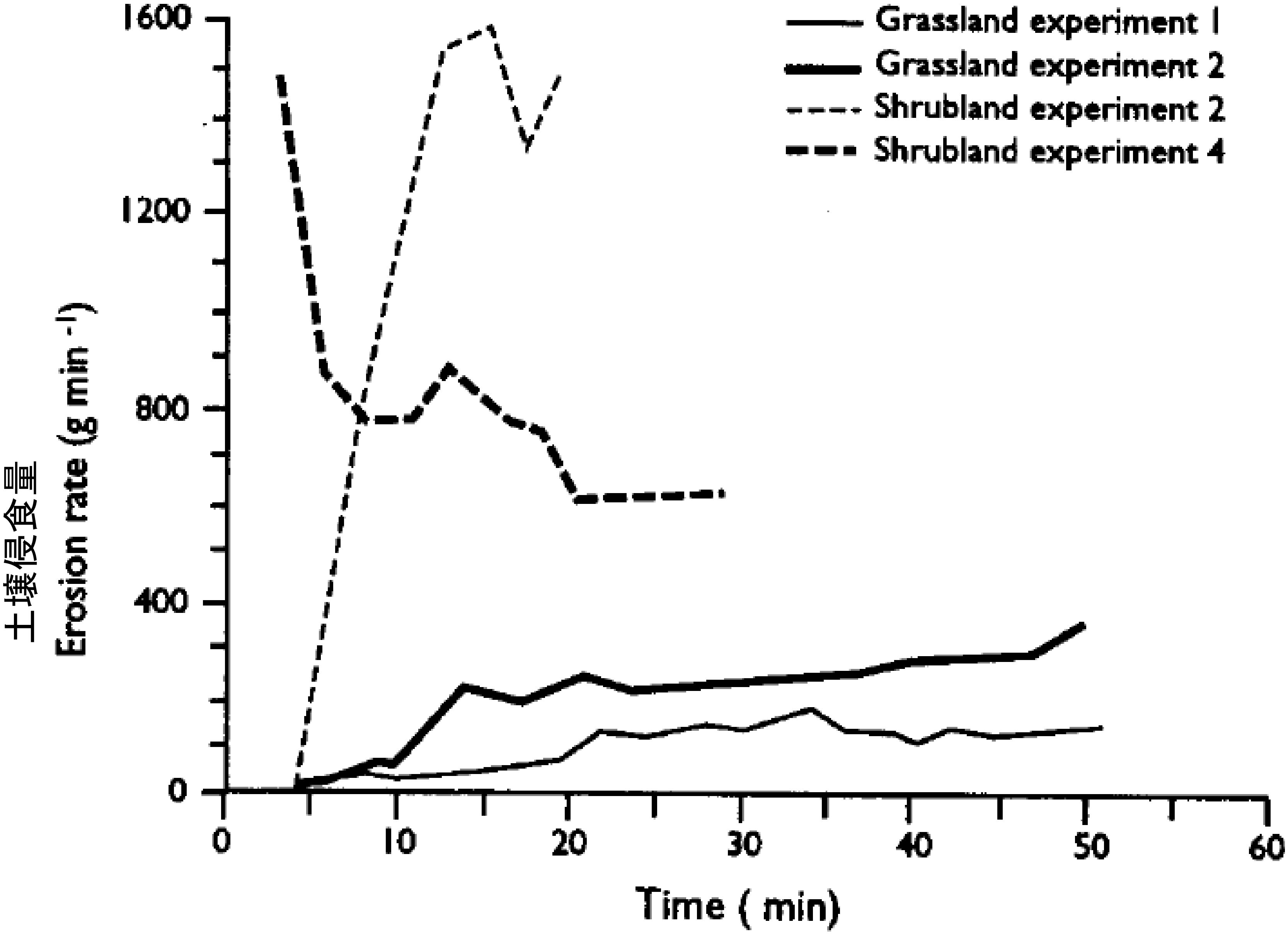




Rainfall energy
Soil strength variability
unrilled
rilled
Flow energy



Grassland experiment 1
Grassland experiment 2
Shrubland experiment 1
Shrubland experiment 2
Discharge (cm³ s⁻¹ m⁻²)
流出量
15
10
5
0
0
10
20
30
40
50
60
Time (min)



Grassland experiment 1
Grassland experiment 2
Shrubland experiment 2
Shrubland experiment 4
1600
1200
800
400
0
土壤侵食量
Erosion rate (g min -1)
0
10
20
30
40
50
60
Time (min)

Gullies

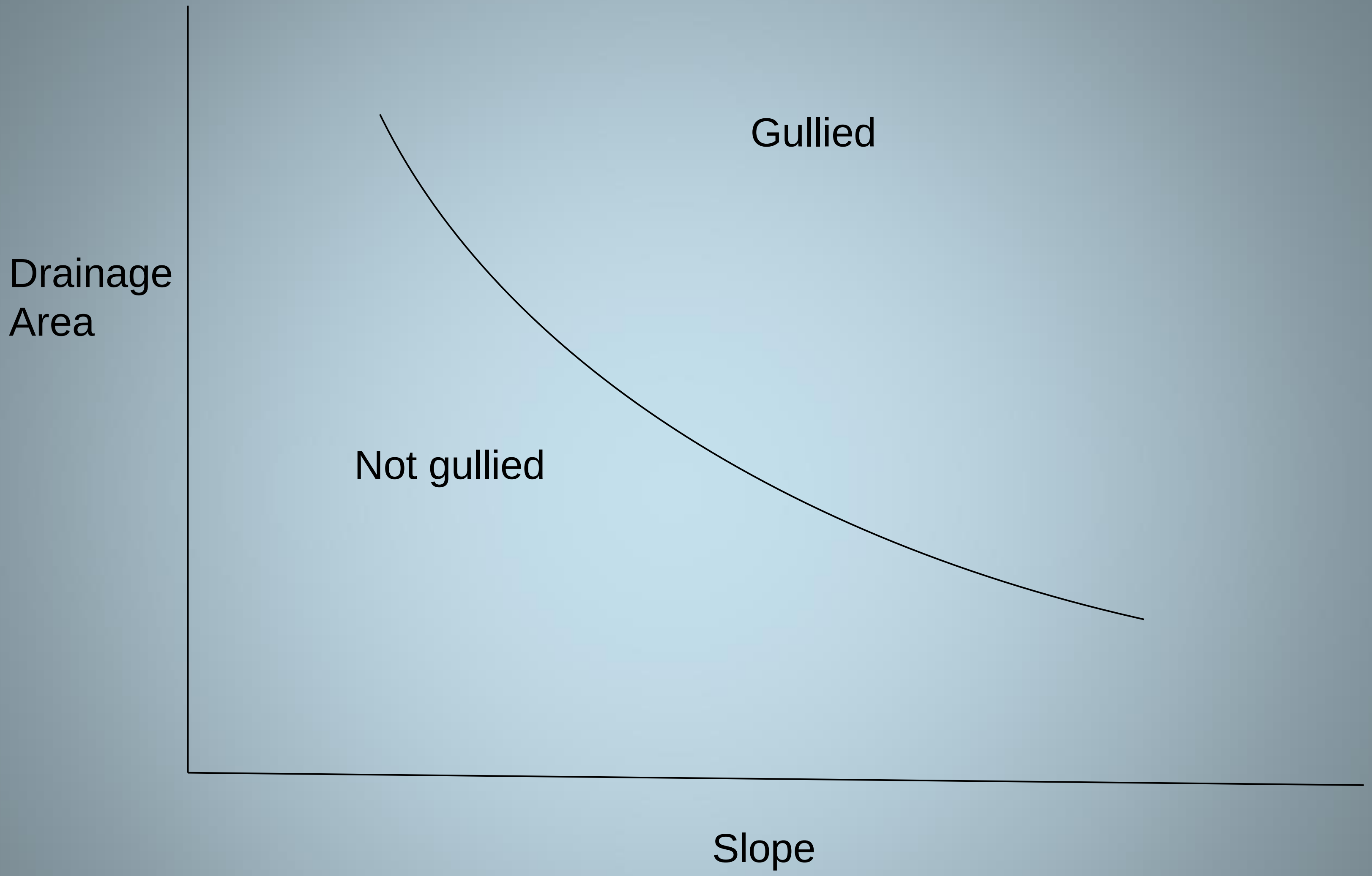
Begin & Schumm (1979):

$$\tau_{cr} = (c\gamma)A^{rf}S$$

（集水面積）　　（傾斜角）

Where A is drainage area (ha), S is slope, rf is an exponent, c is a constant and γ is the weight of water per unit volume

$$S = aA^{-b}$$


Gullied
Drainage Area
Not gullied
Slope